ONDAS DE PARTÍCULAS (ELÉTRONS)

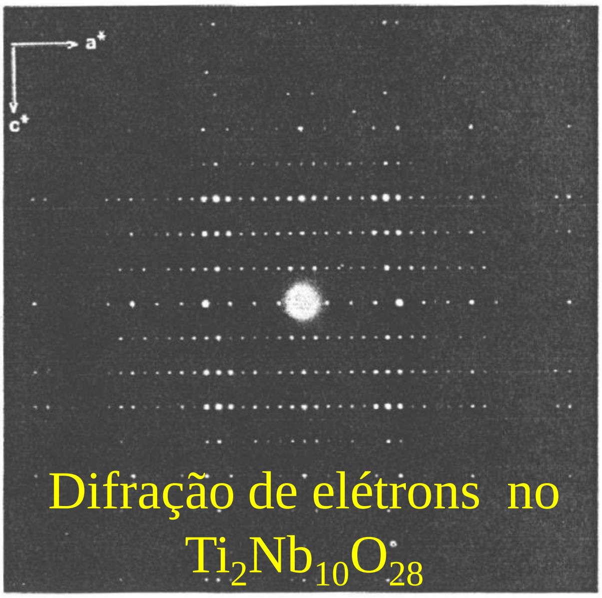


FIGURE 4.2 An electron diffraction pattern. Each bright dot is a region of constructive interference, as in the X-ray diffraction patterns of Figures 3.7 and 3.8. The target is a crystal of $Ti_2Nb_{10}O_{29}$. (Courtesy of Sumio Iijima, Arizona State University.)

FIGURE 4.3 Comparison of X-ray diffraction and electron diffraction. The upper half of the figure shows the result of scattering of 0.071 nm X rays by an aluminum foil, and the lower half shows the result of scattering of 600 eV electrons by aluminum. (The wavelengths are different so the scales of the two halves have been adjusted.) (*Source:* Education Development Center, Newton, MA.)

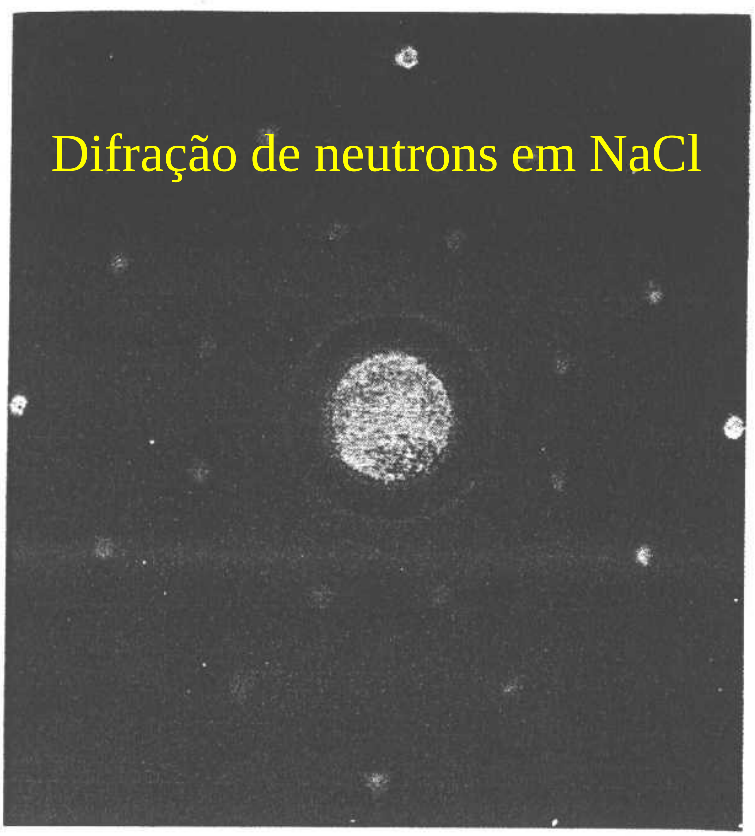


FIGURE 4.4 Diffraction of neutrons by a sodium chloride crystal. (*Source:* Eisberg & Resnick, *Quantum Physics*, John Wiley & Sons, 1974.)

ONDAS DE PARTÍCULAS (NEUTRONS)

Exp. de Davisson-Germer ⇒ provar as ondas de elétrons de De Broglie

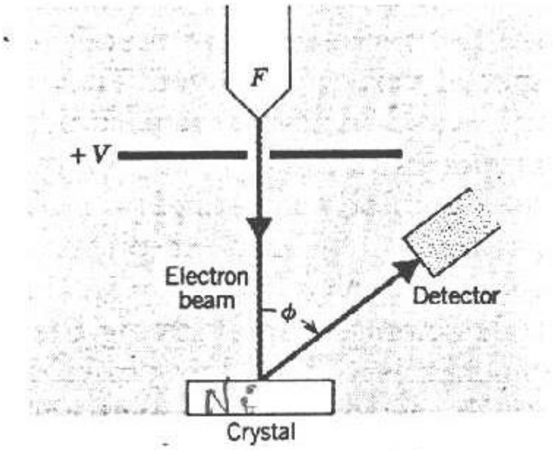


FIGURE 4.6 Apparatus used by Davisson and Germer to study electron diffraction. Electrons leave the filament *F* and are accelerated by the voltage *V*. The beam strikes a crystal and the scattered beam is detected at an angle ϕ relative to the incident beam. The detector can be moved in the range 0 to 90°.

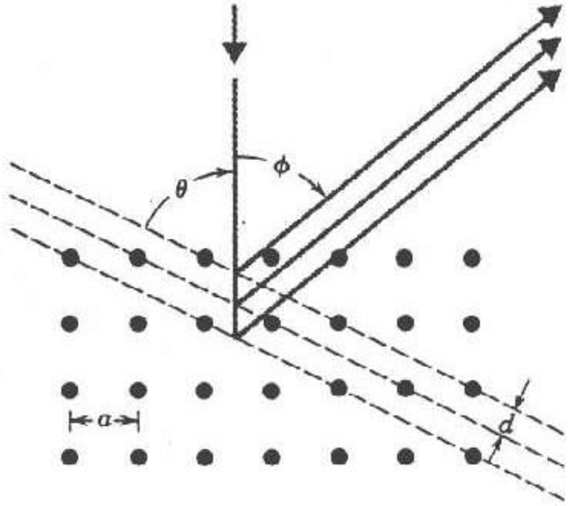


FIGURE 4.7 Detail of scattering from crystal planes. The atoms in the crystal are separated by a distance *a*, and at the Bragg angle θ the spacing of atomic planes is *d*. Constructive interference of the scattered beams occurs when the Bragg condition is satisfied.

Interferência construtiva

Ângulo de Bragg $\theta = 90^o - \phi/2$ e $\lambda = 2d\mathrm{sen}\theta$

$d = a\,\mathrm{sen}(\phi/2)$

$a(\mathrm{Ni}) = 0.215\mathrm{nm}$

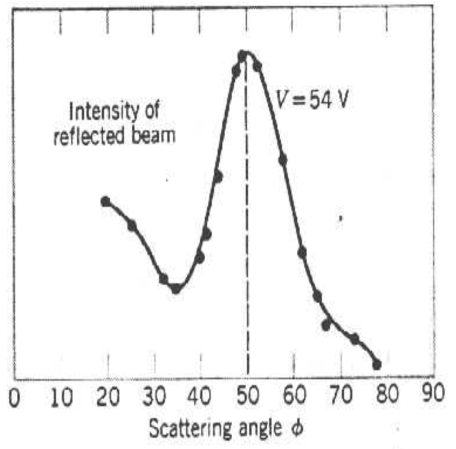


FIGURE 4.8 Results of Davisson and Germer. Constructive interference causes the intensity of the reflected beam to reach a maximum at $\phi = 50°$ for $V = 54$ V.

Interferência construtiva

Intensidade máxima p/ V=54V e ϕ=50º

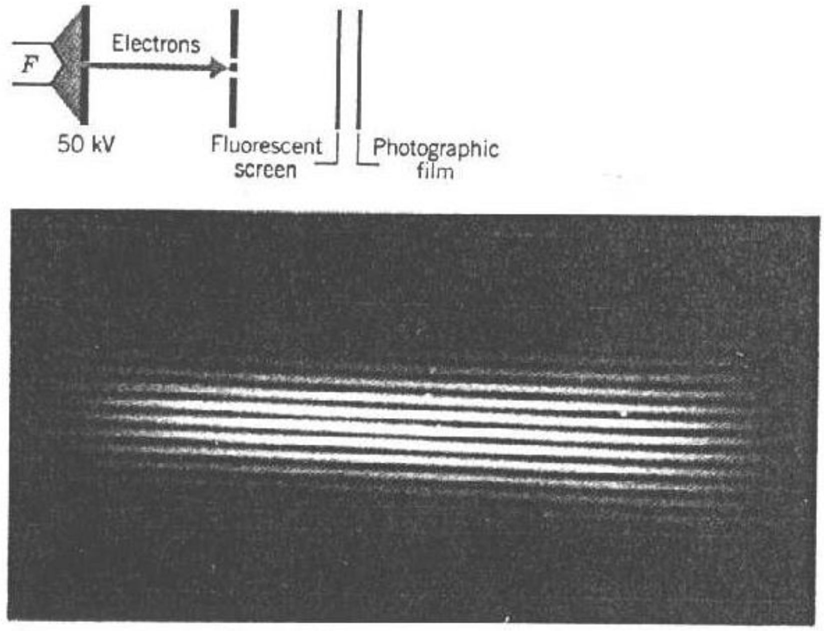
$d = a\,\mathrm{sen}(25^o) = 0.0909\mathrm{nm}$

$\lambda = 2d\mathrm{sen}\theta = 0.165\mathrm{nm}$

De Broglie: $\lambda = h/p$ e $p = (2mk)^{1/2}$ e $k = 54\mathrm{eV}$

λ= 0.167nm

EXP. - DUPLA FENDA -TIPO YOUNG
ONDA DE ELÉTRONS - C. JONSSON



DIFRAÇÃO DE DUPLA FENDA COM ELÉTRONS

FIGURE 4.9 Double-slit diffraction with electrons. Electrons from the filament *F* are accelerated through 50 kV and pass through the double slit. They produce a visible pattern when they strike a fluorescent screen (like a TV screen) and the resulting visual pattern is photographed. The resulting interference pattern is shown. (*Source:* C. Jönsson, *Am. J. Phys.* 42, 4 (1974).)

P/ DETECTAR PASSAGEM DE ELÉTRON

ESPIRA EM VOLTA DE CADA FENDA

CAMPO B INDUZIDO DESTRÓI INTERFERÊNCIA

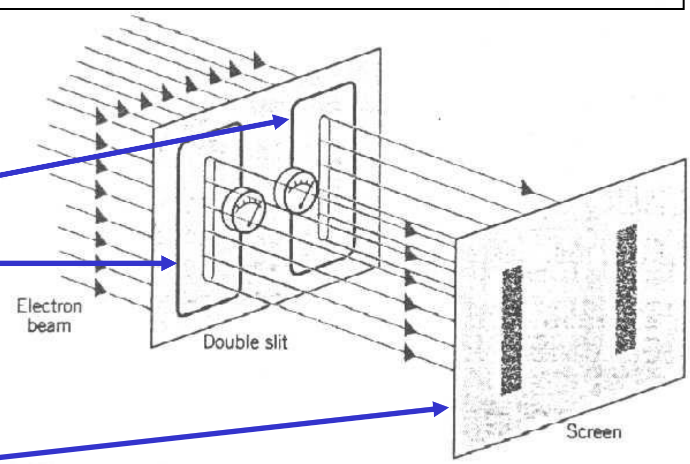


FIGURE 4.10 Apparatus to record passage of electrons through slits. Each slit is surrounded by a loop of wire with a current meter that signals the passage of an electron through the slit. No interference finges are seen on the screen.

∴ NÃO SE PODE MEDIR OS COMPORTAMENTOS CORPUSCULAR E ONDULATÓRIO

PRINCÍPIO DA COMPLEMENTARIDADE

Princípios de incerteza das ondas clássicas

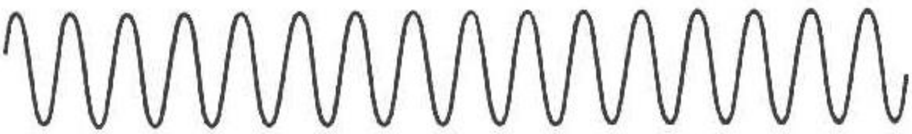


FIGURE 4.11 A pure sine wave, which extends from $-\infty$ to $+\infty$.

$y = y_1 \operatorname{sen} k_1 x$

$k = 2\pi/\lambda$

número de onda

$y = y_1 \operatorname{sen} k_1 x + y_2 \operatorname{sen} k_2 x$ → superposição

Onda se localiza em $\approx \Delta x$

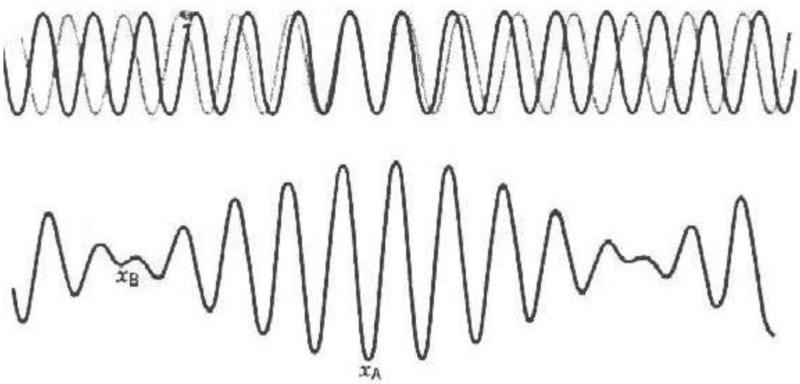


FIGURE 4.12 The superposition of two sine waves of nearly equal wavelengths to give beats. The two sine waves differ in wavelength by 10 percent but have the same amplitude.

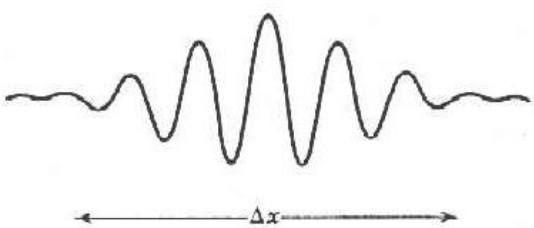


FIGURE 4.13 The resultant of the addition of many sine waves (of different wavelengths and possibly different amplitudes.)

$\Delta x \, \Delta k \approx 1$

1º princípio de incerteza das ondas clássicas

$$\boxed{\Delta x \, \Delta \lambda \approx \lambda^2}$$

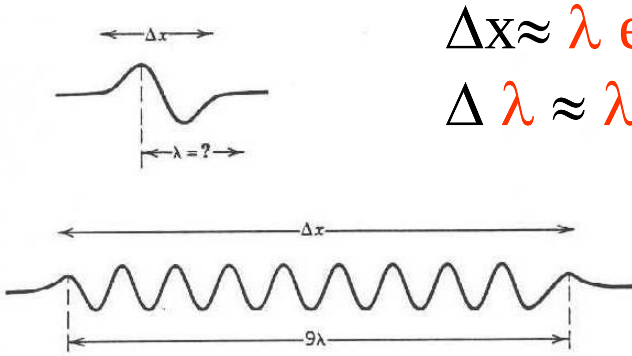


FIGURE 4.14 Two different groups of waves.

$\Delta x \approx \lambda$ e

$\Delta \lambda \approx \lambda$

$\Delta x \approx N\lambda$ e

$\Delta \lambda \approx \lambda / N$

2º princípio de incerteza das ondas clássicas

$$\boxed{\Delta t \, \Delta \omega \approx 1}$$

Partícula entre duas paredes (uma dimensão)

•Início: paredes em $\pm\infty \Rightarrow$partícula em repouso $p_x=0$
e $\Delta p_x=0$ e $\Delta x=\infty$
qualquer solução é possível

•paredes em $\pm L/2 \Rightarrow$separadas por L $\Rightarrow$ $-L/2 \leq x \leq +L/2$
é possível posição $\Delta x \approx L$ e $\Delta p_x \approx$
⇗/L

Conseqüência - Princípio de Incerteza de Heisenberg

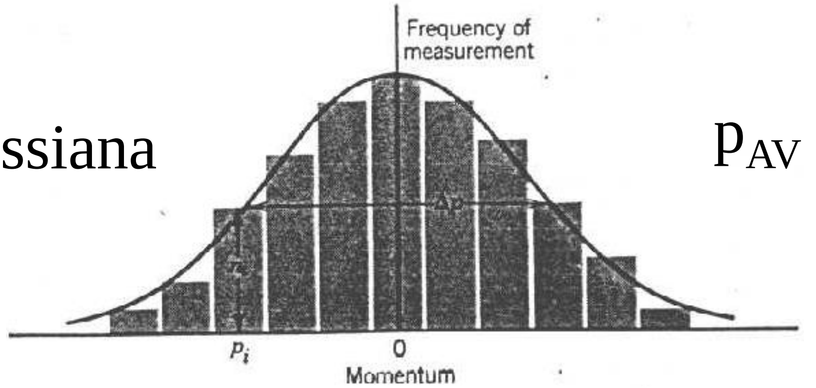
Uma partícula restrita a um espaço finito não pode ter energia cinética nula

•paredes em $\pm L/2 \Rightarrow$ pode-se obter $p_x \neq 0$
$\Rightarrow$ porém medidas em torno de $p_{AV} = 0$.

•Com os desvios de Δx e $\Delta p_x \Rightarrow$ obtém -se função distribuição de medidas p_x onde:

$$\Delta p = \sqrt{(p^2)_{AV} - (p_{AV})^2}$$

Dist. gaussiana



$p_{AV} = 0$

FIGURE 4.15 The momentum of a particle confined to a region Δx. Repeating the measurement many times, each value p_i is measured n_i times. The average momentum is zero, and the distribution has a width $\Delta p \sim \hbar/\Delta x$.

Princípio de incerteza de Heisenberg -Ex.2

Feixe de elétrons passa por uma fenda de abertura *a*, com direção y e momento p_y ($p_x = 0$)

Pelo Princípio de incerteza de Heisenberg:

- antes da fenda: p_y = cte, $p_x = 0$, $\Delta p_x = 0$ e $\Delta x = \infty$.
- ao passar pela fenda: $\Delta x \approx a$ e $\Delta p_x \approx \hbar / a$

$\therefore$ ao passar pela fenda - as partículas ganham p_x de valor $\approx \hbar / a$

Por difração:

Primeiro mínimo por difração

$\lambda = a \,\text{sen}\,\theta$

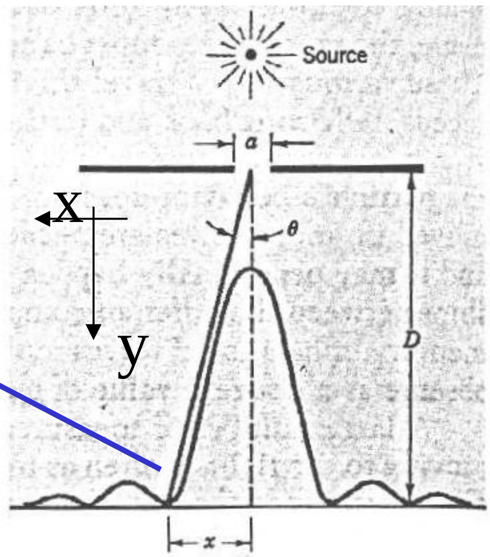


FIGURE 4.16 The intensity observed in single-slit diffraction. The slit has width *a* and the screen is at a distance *D* from the slit. The first diffraction minimum is a distance *x* from the center.

P/ θ pequeno, $\text{sen}\,\theta \approx tg\,\theta = x/D$
e por De Broglie, $\lambda = h/p_y$

$a(x/D) = h/p_y$

$$\boxed{\text{Como } x/D = p_x / p_y \Rightarrow p_x = p_y\,(x/D) = h/a}$$

$\therefore$ as duas descrições são equivalentes p/ p_x de valor $\approx \hbar / a$

Pacotes de onda

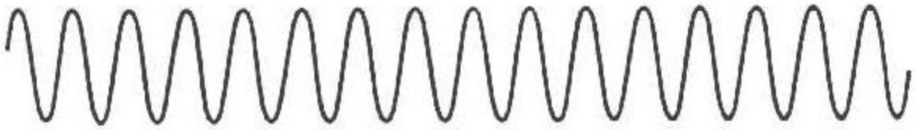


FIGURE 4.11 A pure sine wave, which extends from $-\infty$ to $+\infty$.

$y = A\cos k_1 x$

$k = 2\pi/\lambda$

número de onda

Pacote de onda ideal

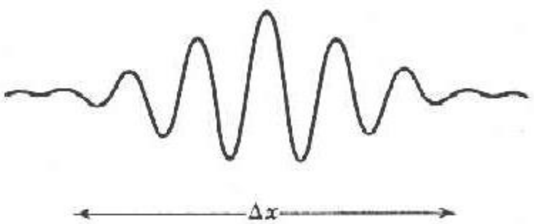


FIGURE 4.13 The resultant of the addition of many sine waves (of different wavelengths and possibly different amplitudes.)

Corresponde a partícula localizada na região $\Delta x \Rightarrow \Delta p_x \approx \hbar / \Delta x$, com correspondente $\Delta\lambda$ ou à superposição de um número de ondas com vários λ

Superposição (batimento de ondas) $\rightarrow$ 2 ondas com k_1 e $k_2 = k_1 + \Delta k$

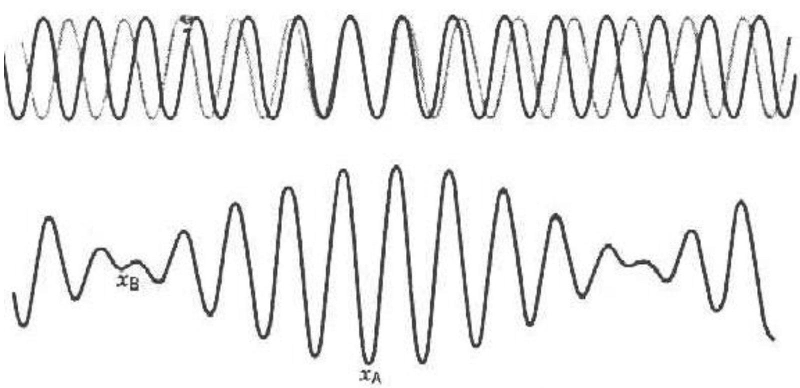


FIGURE 4.12 The superposition of two sine waves of nearly equal wavelengths to give beats. The two sine waves differ in wavelength by 10 percent but have the same amplitude.

$$y = A\cos k_1 x + A\cos k_2 x$$

$$= \underbrace{2A\cos(\Delta k x/2)}_{\text{Envelope de variação de amplitude}} \cos[(k_1 + k_2)x/2]$$

Pacotes de onda

Para ondas caminhantes (dependentes do tempo) substitui-se **kx** por **(kx-ωt)**:

ω = freq. angular;
$v_{ph} = \omega/k$ = velocidade de fase da onda
= veloc. de propagação de 1 onda

Do batimento de 2 ondas:

$$y = A\cos(k_1x - \omega_1 t) + A\cos(k_2 x - \omega_2 t)$$

$$= \underbrace{2A\cos[(\Delta kx - \Delta\omega t)/2]}_{\textit{Envelope de variação de amplitude}}\cos\{[(k_1+k_2)x]/2 - [(\omega_1+\omega_2)t]/2\}$$

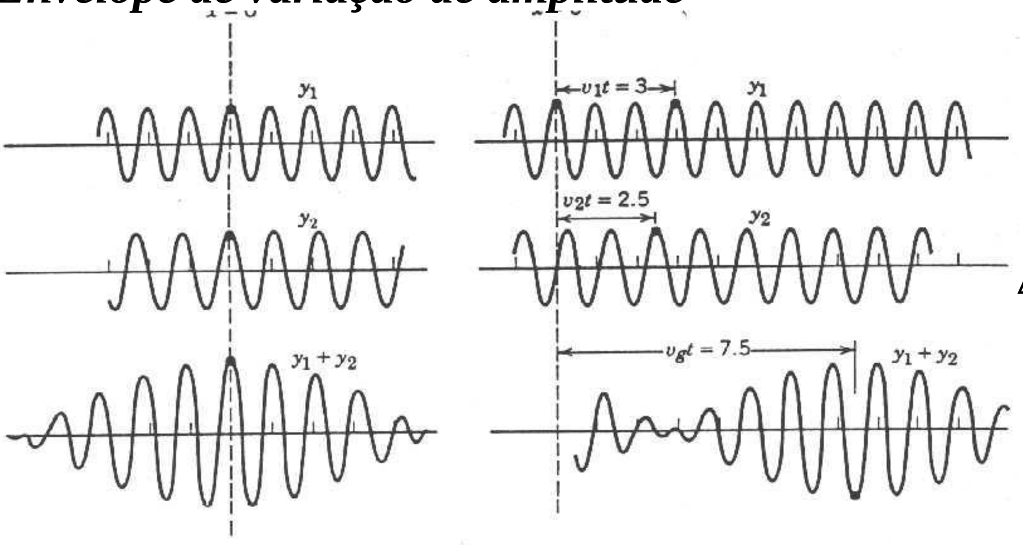


FIGURE 4.18 The group speed of a wave packet. At left is shown a "snapshot" at $t = 0$ of the waves y_1 and y_2 and their sum (y_1 has wavelength 1 unit and y_2 has wavelength $\frac{6}{5}$ unit.) Wave 1 moves with speed 3 units per second and wave 2 with speed 2.5 units per second. A snapshot at $t = 1$ s is shown at right. The two waves are not in phase until the point at 7.5 units, so the midpoint of the "beat" moves at a group speed of 7.5 units per second, in this case much greater than v_1 or v_2.

Onde:
$\Delta k = k_2 - k_1$
e
$\Delta\omega = \omega_2 - \omega_1$

$\therefore$ o envelope se move c/ vel. $v_g = \Delta\omega/\Delta k$ e a onda dentro do envelope c/ vel. $v_{ph} = (\omega_1 + \omega_2)/(k_1 + k_2)$

Se Δk e $\Delta\omega \downarrow \Rightarrow v_{ph} = v_{ph1} = v_{ph2}$
e $v_g = \Delta\omega/\Delta k$ pode ser $\neq$ v_{ph}

Pacotes de onda

A soma de 2 ondas difere de um pacote de ondas (necessita-se de mais ondas):

$$y(x) = \sum_{ki} A(ki) \cos kix \qquad \text{ou} \qquad y(x) = \int_{k1}^{k2} A(k) \cos kx\,dk$$

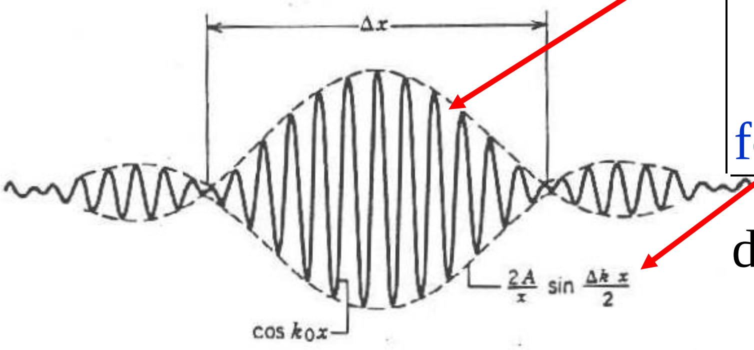
Caso k variar de (k_o-Δ k/2) à (k_o+Δ k/2) c/ A=cte, obtém-se da integral:

$$\boxed{y(x) = \frac{2A}{x} \text{sen}\left(\frac{\Delta k}{2} x\right) \cos kox}$$

A função $\cos kox$ oscila dentro do envelope

$\frac{2A}{x} \text{sen}\left(\frac{\Delta k}{2} x\right)$ próximo do formato do pacote de onda

de acordo c/ o princípio de incerteza p/ $\Delta x \downarrow \rightarrow \Delta k \uparrow$



Melhor aprox. p/ pacote: (forma gaussiana)

$$A(k) = \exp\left[\frac{-(k-ko)^2}{2(\Delta k)^2}\right]$$

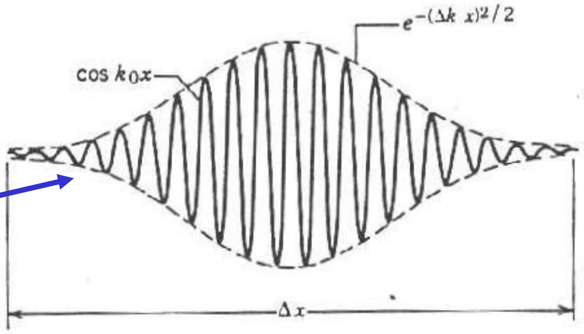


FIGURE 4.19 Example of two different wave packets. In each case there is a modulating function that reduces the amplitude of the cosine beyond the region Δx.

$$y(x)\ \alpha\ \exp\left[\frac{-(\Delta kx)^2}{2}\right] \cos kox$$

Pacotes de onda

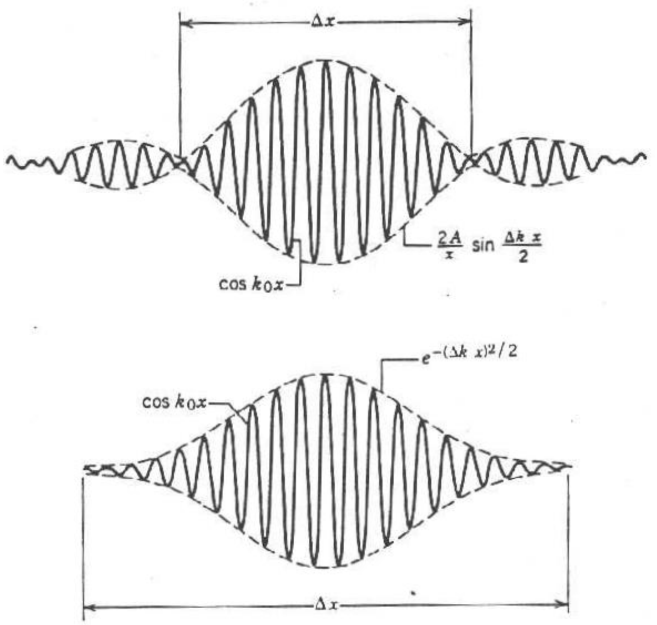


FIGURE 4.19 Example of two different wave packets. In each case there is a modulating function that reduces the amplitude of the cosine beyond the region Δx.

As equações e as formas de onda são p/ t=0

Para ondas caminhantes (dependentes do tempo) substitui-se **kx** por **(kx-ωt)**:

ω = freq. angular;

$v_g = \Delta\omega/\Delta k$ = velocidade do grupo ou envelope;
e vg é a velocidade do elétron = p/m

$v_{ph} = \omega/k$ = velocidade de fase da onda de cada componente dentro do grupo

5. PROBABILIDADE

•Suponha preparar 1 átomo, adicionando-se 1 elétron. Onde estará este elétron após Δt?

↓

Repetindo-se este experimento

↓

Obtêm-se resultados diferentes

↓

Distribuição estatística

•Teoria matemática da Mecânica Quântica

↓ permite calcular

Média ou probabilidade de resultados de um experimento e a distribuição de um grande número de resultados

6. AMPLITUDE DA PROBABILIDADE

• A localização de 1 partícula $\longrightarrow$ 1 pacote de ondas

Função tem maior amplitude $\longrightarrow$ Δx $\longrightarrow$ Onde se localiza a partícula

$\therefore$ $\uparrow$amplitude da onda $\Rightarrow$ onde a partícula tem $\uparrow$probabilidade de ser encontrada

A amplitude de onda de De Broglie

relacionada

probabilidade de ser encontrada a partícula em Δx

$$P(x)dx = \psi^2 dx$$

Amplitude A de De Broglie $\Rightarrow$ quadrado absoluto $|A|^2 \alpha P(x)$

A amplitude pode ser um número complexo

Seu quadrado absoluto é real

$$A^* A$$